COMPARISON BETWEEN FDTD GRADED GRIDS

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Abstract

The traditional FDTD discretization of the Maxwell equations in which all cells are equal has been proved not to be efficient in many cases. These cases include the structures in which the field intensities are not uniformly distributed in the computation domain. However the direct use of graded discretization grids inserts a first order error in the conventional difference equations. Many authors have proposed solutions for this problem. In the present work, some of these solutions are reviewed, implemented and evaluated. The solution proposed by the authors in a previous paper has proved to give better results in terms of accuracy and stability for greater length ratio between neighbouring cells.

1 Introduction

Since 1966, as Yee proposed his uniform mesh [1] for the finite difference time domain (FDTD) analysis of electromagnetic problems, a lot of advances has been made in order to get the method more efficient. Choi and Hoefer [2] have presented a graded mesh which is able to concentrate the computational efforts in the regions of greater field intensity. This procedure increases the number of computer operations for each cell in each time step, but allows the use of less cells for the same accuracy, which leads to a faster calculation. Xiao and Vahldieck [3] have shown a graded scheme which allows the correction of the first order error. Much more computation is needed for each cell by this method due to the corrections one should make to ensure the maintainance of the second order accuracy. But the strong reduction of the number of cells for a given precision causes a better efficiency than the other schemes. A similar procedure has been introduced by Krupežević *et allii* [4]. In this case the discretization is made using three points instead of two for each differentiation, providing second order accuracy. In a previous paper [5], Tupynambá and Omar have proposed another method for correcting the first order errors by using extrapolation. This procedure has already been proved to give very small reflections even for very large length ratios between neighbouring cells [6].

In this paper, these four different graded grid procedures are compared. In the next section, these procedures are reviewed and a generalized equation is introduced that is able to represent each of them with appropriate change of coefficients. Section 3 presents the numerical results of these methods by the computation of propagation characteristics in a shielded coplanar waveguide using graded grids with different length ratios. A general conclusion is then drawn regarding accuracy and stability of the techniques. This conclusion intends to help the FDTD developer in the choice of the most adequate discretization procedure.

2 Graded grid procedures

In this section, different graded grid procedures are presented. In order to simplify the formulation, only the derivative of H_y in the x-direction is discretized in the different ways. All quotations and points are refered in Fig. 1. The initials of the authors are given for identification of the techniques.

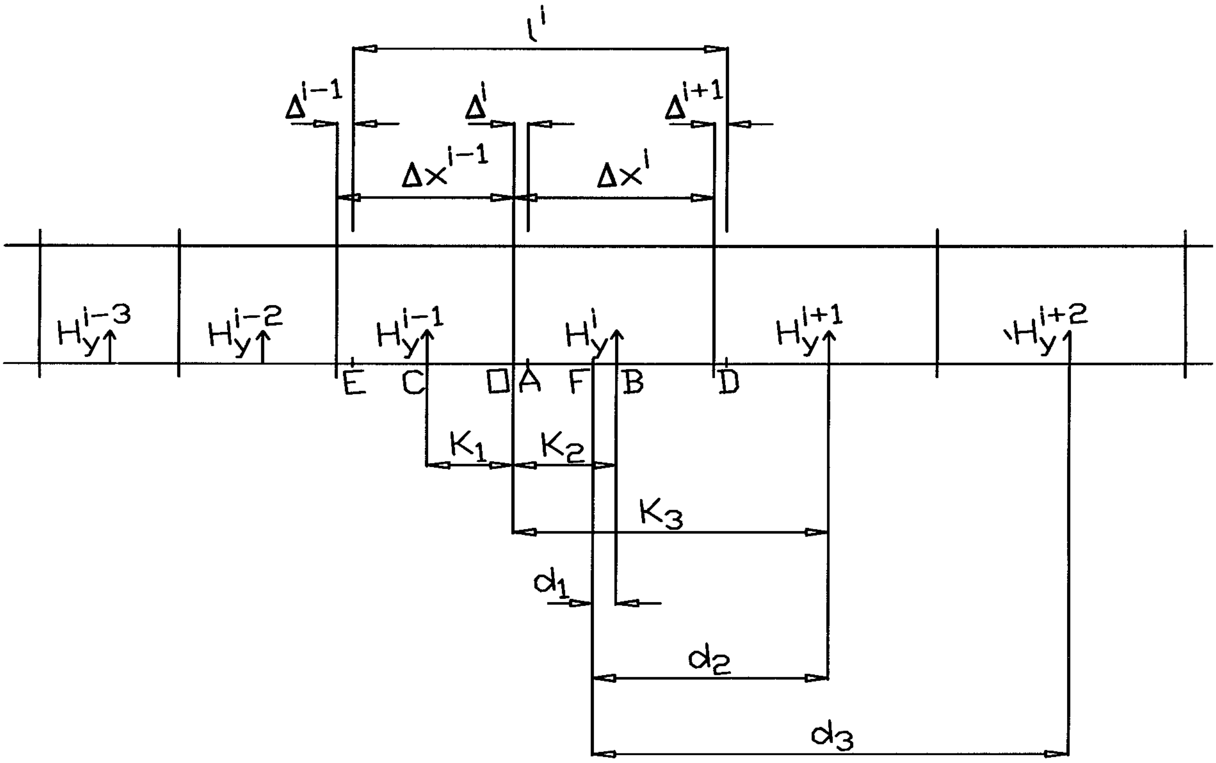

Figure 1: Graded grid in the x-direction

A. Choi and Hoefer procedure (CH)

Choi and Hoefer [2] have made a direct first order discretization of the derivative. This yields

$$\left.\frac{\partial H_y}{\partial x}\right|_O = \frac{H_y^i - H_y^{i-1}}{\frac{\Delta x^i + \Delta x^{i-1}}{2}}. \tag{1}$$

One should observe that, at the central point "O" where E_z is to be calculated (Fig. 1), this equation has a first order error for different values of Δx^i and Δx^{i-1}.

B. Xiao and Vahldieck procedure (XV)

Xiao and Vahldieck [3] have noticed that equation (1) presents second order accuracy at the point "A", which is located at the same distance from the points "B" and "C", where the field component H_y is calculated (see Fig. 1). In the same way, this derivative can be calculated at the points "D" and "E" with second order accuracy.

Using an interpolation of these three values one can obtain the corrected derivative at "O", according to the following equation:

$$\left.\frac{\partial H_y}{\partial x}\right|_O = \left.\frac{\partial H_y}{\partial x}\right|_A - \frac{\Delta^i}{l^i}\left(\left.\frac{\partial H_y}{\partial x}\right|_D - \left.\frac{\partial H_y}{\partial x}\right|_E\right), \tag{2}$$

where Δ^i and l^i are shown in Fig. 1.

C. Krupežević *et allii* procedure (KBA)

Krupežević *et allii* [4] have made the direct second order discretization of the derivative using three points. Using the present notation and calculating for the point "O" results in

$$\begin{aligned}\left.\frac{\partial H_y}{\partial x}\right|_O &= \frac{1}{K_1 + K_3}\left[\frac{K_2 + K_3}{K_1 + K_2}\left(H_y^i - H_y^{i-1}\right)\right. \\ &- \left.\frac{K_2 - K_1}{K_3 - K_2}\left(H_y^{i+1} - H_y^i\right)\right], \end{aligned} \tag{3}$$

where K_1, K_2 and K_3 are shown in Fig. 1.

D. Tupynambá and Omar procedure (TO)

Tupynambá and Omar [5] have considered the possibility that the change of cell length could be accompained by an interface between two media.

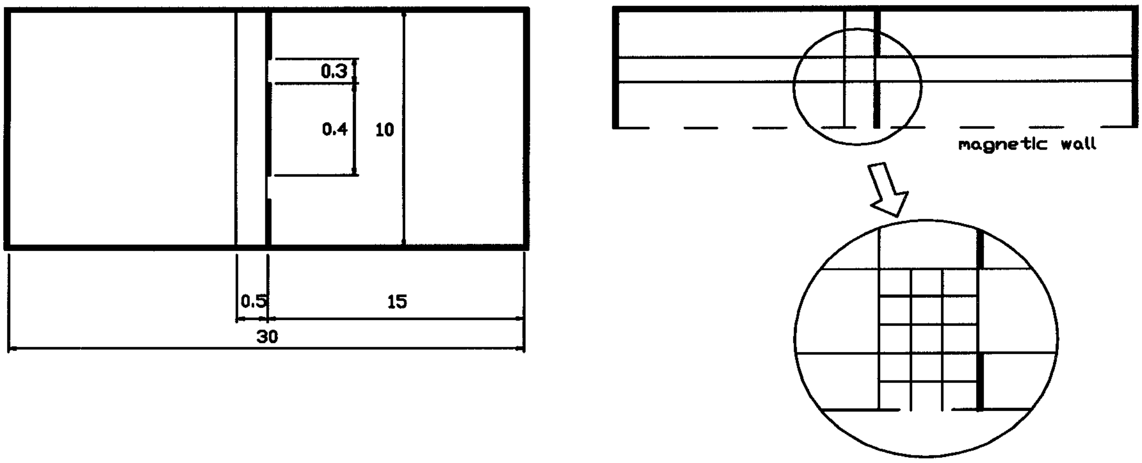

Figure 2: Transversal cut of a shielded coplanar waveguide

In this case, this derivative is not continuous at this interface so that the arguments of second order accuracy do not apply.

They have proposed the extrapolation of H_y at the side where the cell is coarser (right side in Fig. 1) to the point "F", so that "C" and "F" are equidistant from the interface (point "O"). The extrapolation is made using a second order trace of the field component values H_y^i, H_y^{i+1} and H_y^{i+2}, which results in

$$H_y|_F = \frac{1}{d_3 - d_1}\left(d_3 \frac{d_2 H_y^i - d_1 H_y^{i+1}}{d_2 - d_1} - d_1 \frac{d_3 H_y^{i+1} - d_2 H_y^{i+2}}{d_3 - d_2}\right), \quad (4)$$

where d_1, d_2 and d_3 are shown in Fig. 1.

The derivative can then be expressed in the usual form:

$$\left.\frac{\partial H_y}{\partial x}\right|_O = \frac{H_y|_F - H_y^{i-1}}{\Delta x^{i-1}}. \quad (5)$$

A similar set of equations can be found for the case where the coarser cell is on the left hand side, that means $\Delta x^{i-1} > \Delta x^i$.

E. Generalized equation

All these four procedures can be reduced to the following generalized equation:

$$\left.\frac{\partial H_y}{\partial x}\right|_O = \sum_n C_n H_y^{i+n}, \quad (6)$$

where C_n can be calculated for each case using the above equations (1) to (5).

These coefficients are not presented here due to space restrictions. It is however important to notice that CH needs only two coefficients, KBA needs three, and XV and TO each four. TO needs furthermore a decision on which cell is longer. The number of coefficients will determine the computation time for each cell, but not for the whole structure. This depends on the number of cells that must be included, which depends on the performance of the used technique. This performance will be evaluated in the next section.

3 Numerical results

For the comparison between these different techniques, the shielded coplanar waveguide shown in Fig. 2 has been calculated using the compact 2D–FDTD [5]. The dielectric substrate of the structure has a relative permitivity $\epsilon_r = 12.5$. Only one half of the structure has been discretized by making use of a magnetic wall at the symmetry plane.

A first calculation has been made in which all cells had the same size, namely 0.167 mm x 0.1 mm in a 180 x 50 grid. This yields obviously equal results for all procedures. This result has been used as reference for the comparison.

In the next calculations, the inner 3 x 5 cells shown in datail in Fig. 2 have been kept unchanged while the surrounding cells have been chosen wider and wider in the directions onto the metallic walls. In each of these directions, i.e. "x", "-x" and "y", going from this inner block, the cells have been grouped in three rows (or columns) in which the cell lengths have been "r"

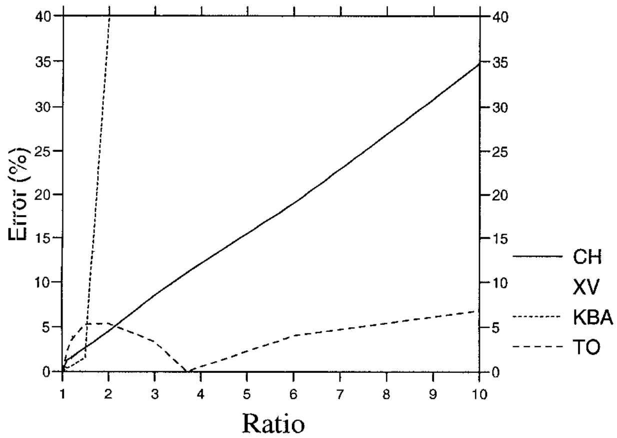

Figure 3: Calculation error in function of the cell length ratio

times the cell lengths of the preceeding group, where "r" is the parametric ratio of each calculation. A limit of $\lambda/20$ at 8 GHz has been fixed for the cell lengths in order not to cause unstability up to this frequency. The ratios have assumed the values 1, 1.1, 1.2, 1.5, 2, 3, 5, 10. The compact 2D-FDTD has been applied with propagation constant β varying from 0.1 to 0.4 rad/mm. The average phase velocity in the resulting frequency range has been used for comparison. Fig. 3 shows the percentual error of the phase velocity compared to the phase velocity of the first calculation (ratio 1) for the different procedures.

KBA and XV have shown good values for very small ratios, but have presented unstability for ratios greater than 1.5. CH had the expected behaviour of a procedure with first order error with nearly linearly increasing error. TO has not presented the best results for small ratios but has maintained the error less then 7 % even for ratio 10.

Further calculations with TO using smaller inner cells and ratio till 20 have shown no further degradation of the relative error. This shows that the error is due to the great length of the outer cells, rather than due to the length step.

4 Conclusions

Four graded grid procedures have been compared by calculating the same structure with different cell length ratios. As expected, CH fails to present good results for greater cell length ratios because it does not include any correction for this numeric discontinuity. On the other hand, KBA and XV were not correct in using interpolation of the derivative of the magnetic field in a region where it is not continuous, namely at the interface between different media.

The procedure proposed by the authors in a previous paper has been the only one of the tested techniques which has presented good performance by cell length ratios greater than 2. Other calculations with this technique have shown a reflection of less than -50 dB due to a cell length ratio of 10. These feature enable the possibility of using a much smaller number of cells in the calculation. In the example used in the last section just 20 x 14 cells have been required in the last run instead of the initial 180 x 50.

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